Assignment on Matrices and Determinants

Instructions

- Answer all questions.
- Show all necessary calculations and justifications.
- Submit your solutions by the due date.

Questions

Addition and Multiplication of Matrices

1. Given matrices
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$
Compute $A + B$ and $A - B$.

2. Given
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$
Compute AB and BA. Is matrix multiplication commutative?

3. Find the product of the matrices:
$$A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 2 \\ 1 & -3 \end{bmatrix}$$

4. If A and B are two matrices such that $AB = BA = I$, what can you say about matrices A and B?

Vector Multiplication

1. Compute the matrix-vector product:

$$A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

2. If A is an $m \times n$ matrix and $\mathbf{v}$ is an $n \times 1$ vector, what is the dimension of $A\mathbf{v}$?

Transpose of a Matrix and Its Properties

1. Find the transpose of the matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

2. Prove that $(A^T)^T = A$ for any matrix A.

3. Verify that $(A + B)^T = A^T + B^T$ for the matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Symmetric and Skew-Symmetric Matrices

1. Determine whether the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 5 & 2 \\ -1 & 2 & 4 \end{bmatrix}$$

is symmetric or skew-symmetric.

2. Show that for any square matrix A, $A + A^T$ is always symmetric and $A - A^T$ is always skew-symmetric.

Minors, Cofactors, and Determinants

1. Compute the minor of element a_{23} in the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

2. Find the determinant of

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & 5 \\ -2 & 1 & 3 \end{bmatrix}$$

3. Show that the determinant of a triangular matrix is the product of its diagonal elements.

4. Compute the cofactor matrix of

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 4 & 2 \\ 0 & 5 & 3 \end{bmatrix}$$

Properties of Determinants

1. Prove that if two rows (or columns) of a determinant are interchanged, the sign of the determinant changes.

2. Compute the determinant of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

   What do you observe? Why?

3. If A is an invertible matrix, show that $\det(A^{-1}) = \frac{1}{\det(A)}$.

Adjoint and Inverse of a Matrix

1. Find the adjoint of

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$$

2. Using the determinant and adjoint, find the inverse of

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

3. If A is an invertible matrix, show that $(A^{-1})^T = (A^T)^{-1}$.

4. If A is a 3×3 matrix such that $A \cdot \text{adj}(A) = \det(A)I$, prove that $A^{-1} = \frac{\text{adj}(A)}{\det(A)}$ when A is invertible.