

Worksheet  
Power Method and Orthogonal projection

Name and section: \_\_\_\_\_

Instructor's name: \_\_\_\_\_

1. Find the dominant eigenvalue and corresponding eigenvectors of the matrix  $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ .
2. Apply the power method  $v_{k+1} = Av_k$  to problem (1) and perform six iterations with initial guess  $v_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ . What is the value of limiting vector  $v_\infty$ ?

3. Find the first three iterations obtained by the Power method applied to matrix  $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ ; Use

$$x_0 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

4. Also, solve the problem (3) for smallest eigenvalue and corresponding eigenvector using the Inverse Power method.
5. Find the orthogonal projection  $\vec{x}^{\parallel} = \text{proj}_v(\vec{x})$  of the vector  $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ , onto vector  $\vec{v} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ .

6. Find the orthogonal projection of  $\begin{bmatrix} 9 \\ 0 \\ 0 \\ 0 \end{bmatrix}$  onto the subspace of  $\mathbb{R}^4$  spanned by

$$\begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -2 \\ 2 \\ 0 \\ 1 \end{bmatrix}.$$