Some remarks on the convergence of the Bisection method

disadv.

Convergence to the root p of f is v. slow

adv.

the method/iterates <u>always</u> converge to the root of $f(x)$.

Proof :)

<u>Th^m (2.1)</u> Let $f \in C[a,b]$ and $f(a)f(b)<0$. The Bisection method generates a sequence $\{p_n\}_{n=1}^{\infty}$ that converges to p as $n \to \infty$; with

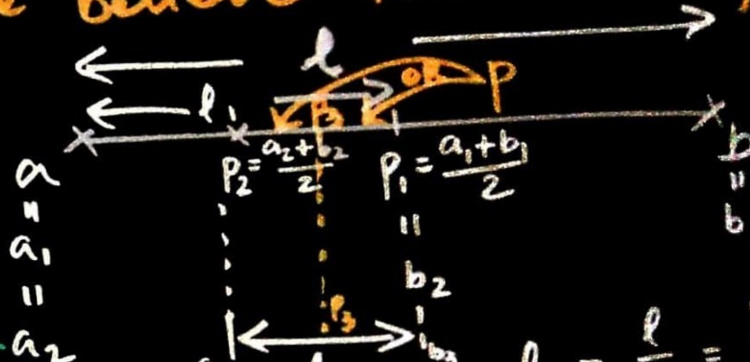
$$|p_n - p| \leq \frac{b-a}{2^n} \quad ; \quad n \geq 1$$

Clearly as $n \to \infty$ R.H.S. $\to 0$!

Proof :-

Notice that $b-a=l$ is the length of the initial domain we are looking for the root of $f(x)$.

Likewise, $(b_n-a_n)=l_n$ is the length of the "shrunk" or "reduced" domain, after n iterations of the Bisection method, where we believe the root of $f(x)$ is!

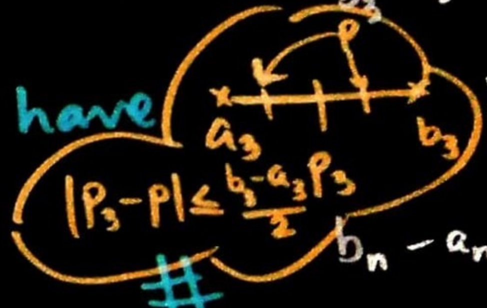


After every iteration the length of the "reduced" domain is halved.

$b_2-a_2=l_1=\frac{l}{2}=\frac{b-a}{2}$

i.e. $b_n-a_n=\frac{1}{2^{n-1}}(b-a)$ and $p\in(a_n,b_n)$

$b_3-a_3=l_2=\frac{l_1}{2}=\frac{l}{4}=\frac{l}{2^2}$



Further, since $p_n=\frac{a_n+b_n}{2}$ $\forall\, n\ge 1$; we have

$$b_n-a_n=\frac{l}{2^{n-1}}$$

$$|p_n-p|\le\frac{b_n-a_n}{2}=\frac{b-a}{2^n}$$

#

i.e. the rate of conv. of $\{p_n\}$ to p is $O\left(\frac{1}{2^n}\right)$!

Reading Assignment for Bisection method.

i) example 2, pg 50 of textbook Burden & Faires 8th edition.

ii) Also read & study the 2 paragraphs immediately following example 2 on pg 50.

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