Computational Linear Algebra: FM 126 — Total time: 1 hour
Phenomenology assessment for section L2 — May 01, 2025

Full Name: _____ **UID:** _____

Instructions: You must **not** be in possession of any electronic devices like laptops, calculators, and GPT-services while responding to this multiple-choice questionnaire. Answer **all the multiple-choice questions (MCQs)** by circling the correct option. The score allotted to each question is **one**. ***If you mark more than one option as your answer to any question, your response will be treated as incorrect (even if one of the opted answers is the correct answer).*** **Maximum score is 10**.

==========START OF QUESTIONS==========

1. **Identify the different categories (category 1, category 2, category 3) presented in the movie.**
   A. Male, female, asexual
   B. Juvenile, adult, senior
   C. Predator, prey, inhibitor
   D. Vertebrates, amphibians, echinoderms
   E. Fossil inscription, model inscription, anatomical diagrams

2. **Based on your response to question 1, identify the correct relationship between the categories.**
   A. category $i \longrightarrow$ category j represents the creative transitions between fossilized life forms (i) and artistic renditions of them (j) by an artist, upon observation in a museum, for any distinct pair of categories indexed by i, j. Moreover, inspired drawings of anatomical diagrams (k) in an atlas after observing a fossilized inscription (i) in a museum may be also be represented as an $i \longrightarrow k$ transition between the respective categories.
   B. {category i, category j, category k, category l, ...,} represents a collection of distinct co-habitating species, indexed by $i, j, k, l, \ldots$, that are part of the same food cycle chain.
   C. category $i \longleftrightarrow$ category j represents a transformation between distinct biological sexes for any distinct pair indexed by i, j.
   D. category $i \longrightarrow$ category $i+1 \longrightarrow$ category $i+2 \longrightarrow \ldots$, beginning with $i = 1$, represents the biological transitions across different life-cycle stages of one particular living species.
   E. category $i >$ category $j >$ category $k > \ldots$ represents a phylogenetic hierarchy in the evolutionary stages of living beings for distinct categories (across diverse species) indexed by $i, j, k, \ldots$.

3. **With respect to the category relations described in the different response-options of the aforementioned question, the respective rates of transition between the categories (per hour) are given below. Choose the option that is likely to be the most compliant with the phenomenon depicted in the movie. Your response to this question should be guided by the correct response to question 2.**
   A. $R_{\text{male}\longrightarrow\text{female}} = \dfrac{1}{2}$ is the rate of transformation from the category male to female. Likewise, $R_{\text{female}\longrightarrow\text{male}} = \frac{1}{2}$ and $R_{i\longrightarrow\text{ asexual}} = 0$ for any $i = \{\text{male, female}\}$. $R_{\text{asexual}\to\text{asexual}} = 2$.
   B. $R_{\circledast\to 2} = 4$ is the rate at which the category-2 individuals are birthed per category-2 individuals. $R_{2\to\odot} = \frac{1}{3}$ is the rate at which category-2 individuals are hunted down by category-1 individuals. $R_{1\to 3} = \frac{1}{3}$ is the rate at which category-1 individuals mature and wane to become category-3 inhibitors due to ageing, a process known as amensal inhibition. $R_{\circledast\to 1} = 2$ is the rate at which the category-1 individuals are birthed per category-1 individuals. At every epoch, only 25% of the category-3 inhibitors survive. Here the states of birth and death are represented by the symbols $\circledast$ and $\odot$ respectively.
   C. $R_{n<m} = \dfrac{1}{3}$ is the rate of migration from category n to category m, where category m is a species higher in the evolutionary stage than a species in category n.
   D. $R_{1\to 2} = \frac{1}{32}$, $R_{2\to 3} = \frac{1}{8}$ are the rates of transitions between successive life-stages. $R_{1\to\odot} = \frac{7}{32}$, $R_{2\to\odot} = \frac{1}{8}$, $R_{3\to\odot} = \frac{4}{5}$ are the death rates at each life-stage. $R_{\circledast\to 1} = 2$ per category-2 individuals. Here the states of birth and death are represented by the symbols $\circledast$ and $\odot$ respectively.
   E. $R_{1\to 2} = \frac{1}{2}$ is the rate of creation of a model by an artist and $R_{1\to 3} = \frac{1}{4}$ is the rate at which anatomical maps are created in an observatory that houses atlas-makers of life-forms.

4. **Which one of the following mathematical models best represents this phenomenon shown in the movie? The model parameters are given by the correct answer to question 3. So, question 3 should give you additional cue to the correct model of the phenomenon. Consider C_1, C_2, and C_3 as the populations of the respective three categories.**

A.
$$\frac{d\mathbf{x}(t)}{dt} = A\mathbf{x}(t).$$
$\mathbf{x}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \\ C_3(t) \end{bmatrix}$ and $C_1(t)$, $C_2(t)$ & $C_3(t)$ represents the population of the respective categories at time t. The matrix $A = \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ represents the rates of transitions and asexual reproductions.

B.
$$x^{(k+1)} = rx^{(k)}\left(1 - \frac{x^{(k)}}{N}\right),$$
where $x^{(k)} = C_1^{(k)} + C_2^{(k)} + C_3^{(k)}$ represents the total species population at k^{th} epoch, $r = \frac{1}{3}$ is the growth rate of each category, and $N = 100$ is the carrying capacity of the population.

C.
$$\begin{aligned} C_1^{(k+1)} &= C_1^{(k)} \\ C_2^{(k+1)} &= \frac{1}{2}C_1^{(k)} + C_2^{(k)} \\ C_3^{(k+1)} &= \frac{1}{4}C_1^{(k)} + C_3^{(k)} \end{aligned}$$

D.
$$\begin{aligned} C_1^{(k+1)} &= \frac{2}{3}C_1^{(k)} + 2C_1^{(k)} \\ C_2^{(k+1)} &= (C_2^{(k)} - \frac{1}{3}C_1^{(k)}) + 4C_2^{(k)} \\ C_3^{(k+1)} &= \frac{1}{3}C_1^{(k)} + \frac{1}{4}C_3^{(k)} \end{aligned}$$

E.
$$\begin{aligned} C_1^{(k+1)} &= \frac{3}{4}C_1^{(k)} + 2C_2^{(k)} \\ C_2^{(k+1)} &= \frac{1}{32}C_1^{(k)} + \frac{3}{4}C_2^{(k)} \\ C_3^{(k+1)} &= \frac{1}{8}C_2^{(k)} + \frac{1}{5}C_3^{(k)} \end{aligned}$$

5. **Based on the structure of the correct model equations, which of the following best describes the type of the mathematical model of the phenomenon?**

A. Not enough information to decide
B. Linear Model
C. Quasi-linear Model
D. Non-linear Model
E. None of the options
F. Logistic Model

6. **For the correct model of the phenomenon, at what rate the category-1 individuals remain in the same life-cycle stage?**

A. $\frac{3}{4}$
B. None of the options
C. $\frac{1}{8}$
D. $\frac{5}{16}$

E. $\frac{4}{5}$
F. $\frac{5}{32}$

7. **Which of the following is a possible set of eigenvalues for the correct transition matrix of the model in question 4?**
   A. $1,\ -\frac{1}{5},\ \frac{1}{6}$
   B. $1.0,\ -0.4,\ 0.2$
   C. $1.0,\ 0.8,\ 0.2$
   D. $0.9,\ 0.6,\ 0.3$
   E. $1,\ \frac{1}{2},\ \frac{1}{5}$
   F. None of the options

8. **What would be the proportion of the category-2 individuals in the population in the long run?**
   A. $\frac{32}{293}$
   B. $\frac{1}{32}$
   C. $\frac{5}{293}$
   D. None of the options
   E. $\frac{256}{293}$
   F. $\frac{13}{40}$

9. **What is the approximate long-time carrying capacity of the population (comprising all three categories) if there are about 1000 category-2 individuals in the long-run?**
   A. 293
   B. None of the options
   C. 916
   D. cannot be estimated based on information provided
   E. 9160
   F. 12000

10. **What is the condition that will determine if the population of this species will go extinct in the long-run?**
    A. $R_{3\to\odot} \longrightarrow 0$ for the species
    B. The initial population of category-0 individuals is 0
    C. dominant eigenvalue < 1 for the correct transition matrix of question 4
    D. $R_{1\to 2} >> 1$ for the species
    E. dominant eigenvalue $\to \infty$ for the correct transition matrix of question 4
    F. None of the options

==========END OF QUESTIONS==========