Spring 2023

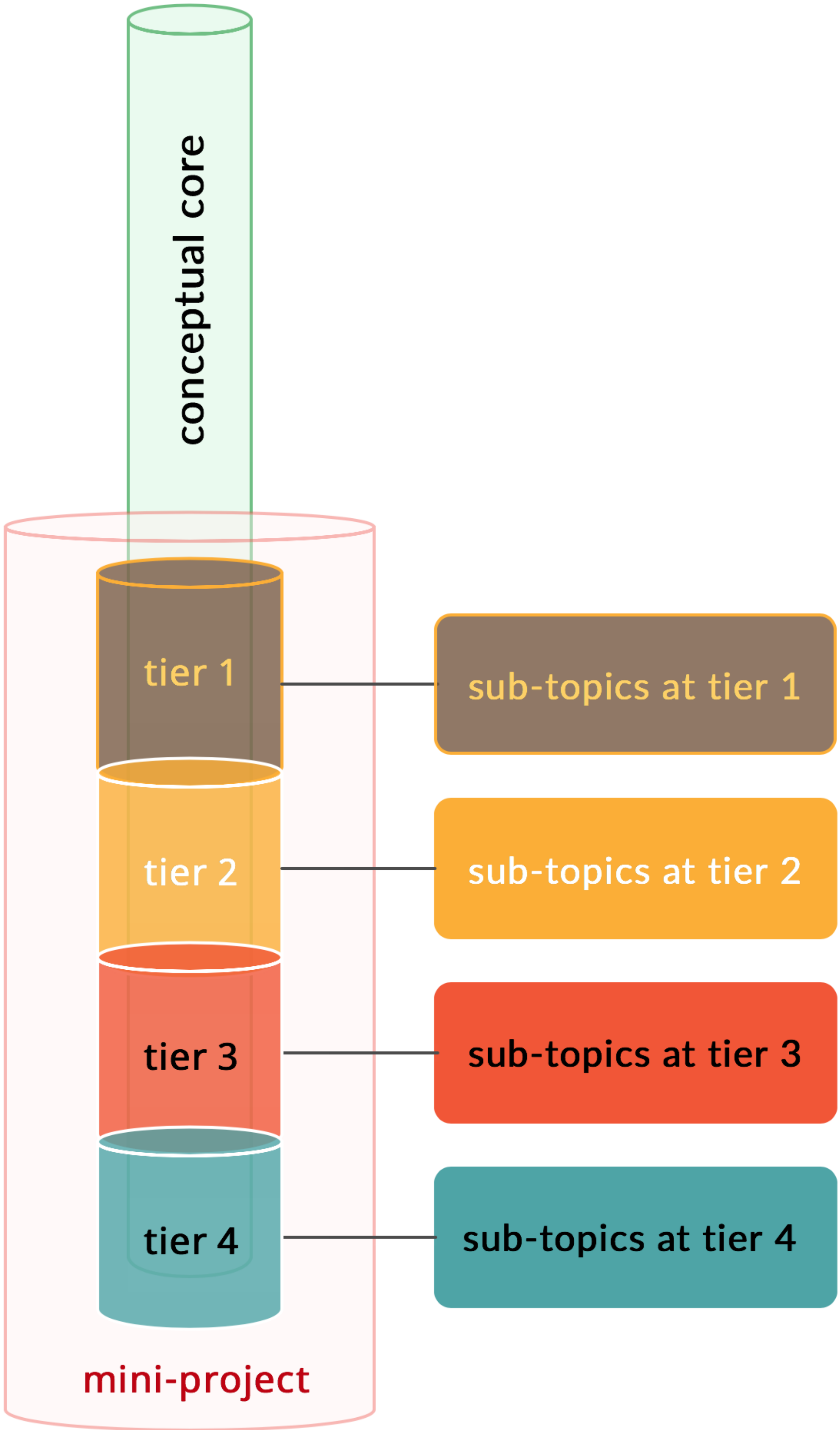
<u>COURSE BROCHURE</u>

Mathematics of Uncertainty

FM 133

Credits = 3 (Lecture: 2, Practice cum laboratory: 1)

Course designed by: *Amrik Sen*



Learning unit

Contents

Chapter 1

Syllabus

1.1 Course preamble

This course will cover fundamental aspects of probability and statistics from the stand point of basic theoretical knowledge and practical applications. Students will acquire training in foundational concepts. Additionally, they will learn how to use a computer to solve diverse engineering problems by building and analysing suitable mathematical models. The course is divided into five modules. Each module comprises a *conceptual core* which is split across multiple sub-modules (tiers). Each tier will cover several related topics that will be discussed over weekly lectures and laboratory classes. There will be two lectures of 50 minutes each, one practice-cum-laboratory class of two hours every week, and one weekly tutorial hour. The course will require completion of two topical projects spread across the semester.

1.2 Pre-requisites

- High school level familiarity with probability and statistics.
- Linear algebra at undergraduate level.
- Experience with a programming language (Matlab or Python).

1.3 Course instructors

- **Lead instructor**: Amrik Sen
- **Teaching Assistants**: Rohit Singla, Abhishek Thakur, Sushma, Viraj D'Souza

1.4 Recommended textbooks

All essential course related materials will be provided on the course website in the form of book chapters, laboratory manuals, and slides (when applicable) and should be sufficient for the learning expected from this course.

- ***Play of Chance and Purpose*** *- An invitation to Probability, Statistics, and Stochasticity using simulations and projects* by *Amrik Sen*, Cambridge University Press, (*in press*).

1.5 Reference books

Interested students may also refer to many classic texts on this subject as extra reference material.

1. ***Weighing the Odds*** *- A course in Probability and Statistics* by *David Williams*, first edition, Cambridge University Press, (2001).

2. ***Probability Theory*** *- The Logic of Science* by *E. T. Jaynes*, first edition, Cambridge University Press, (2003).

3. ***Introduction to Probability and Statistics for Engineers and Scientists*** by *Sheldon M. Ross*, sixth edition, Cambridge University Press, (2021).

4. ***Probability, Random Variables and Stochastic Processes*** by *Athanasios Papoulis and S. Unnikrishna Pillai*, fourth edition, McGraw Hill, (2017).

1.6 Assessments

- One mid-term and one end-term exam (50%, proctured) + two quizes (10%, proctored) = 60% weightage
- Two mini-projects= 30% weightage. Two Interviews = 10% weightage.

1.7 Software

The following programming languages will be used for the laboratory experiments-cum-projects.

- Matlab
- Python

1.8 Course Learning Outcomes (CLOs)

Upon successful completion of this course, students should be able to acquire the following skills.

1. Calculate probabilities and expected outcomes of complex problems based on probability distributions.
Bloom's taxonomy: Remembering, Understanding

2. Compute sampling distributions and identify their asymptotic properties (eg., law of large numbers, central limit theorem)
Bloom's taxonomy: Remembering, Understanding

3. Analyse multi-variate data using regression and principal component analyses.
Bloom's taxonomy: Analyzing

4. Model stochastic processes with finite states as Markov chains and make predictions (eg., random walk).
Bloom's taxonomy: Applying, Analyzing

5. Make decisions under uncertainty in relation to a variety of complex engineering problems.
Bloom's taxonomy: Evaluating, Creating

Chapter 2

Course content and plan

2.1 Topics

Module 1

Conceptual core: Thinking in Probability

(i) **Tier 1:** Definition of probability space and measure, axioms of probability, random variable, conditional probability and the law of total probability.

(ii) **Tier 2:** Bayes' theorem, expected value and variance of a random variable, law of total expectation, law of iterated expectation.

Module 2

Conceptual core: Probability Distributions

(i) **Tier 1:** Meaning of integral with respect to a probability distribution function, definition of cumulative distribution function (cdf), probability mass function (pmf), and probability density function (pdf), different types of discrete and continuous probability distribution functions, compound probability distribution, joint and marginal distribution functions.

(ii) **Tier 2:** Moment generating functions and their applications.

(iii) **Tier 3:** Asymptotic results prescribed by the law of large numbers and the central limit theorem.

Project-1: Predicting insurance claim aggregates during a policy period.

Module 3

Conceptual core: Statistical Experiments

(i) **Tier 1:** Basics of sampling distributions.

(ii) **Tier 2:** Asymptotic distributions for large samples and their properties.

(iii) **Tier 3:** Hypothesis tests, ANOVA.

Module 4

Conceptual core: Fundamentals of stochastic processes

(i) **Tier 1:** Definition of Markov chains, transition probabilities, Chapman-Kolmogorov equations, distribution of states.

(ii) **Tier 2:** Recurring events in Markov chains: hitting probability, return times and absorption times, mean number of returns to a state.

(iii) **Tier 3:** Classification of Markov states and their properties.

Project-2: **Automatic prediction of control laws of an aircraft using the Viterbi algorithm.**

Module 5

Conceptual core: Statistics for Complex Problems: analysis of higher dimensional data

(i) **Tier 1:** Least squares regression.

(ii) **Tier 2:** Basics of multi-variate statistics, stationarity and ergodicity, covariance matrix.

(iii) **Tier 3:** Principal Component Analysis.

2.2 Weekly lecture, laboratory and project plan

A detailed week-wise lecture plan is provided on the course website. The laboratory classes will require access to a computer/laptop and the Matlab/Python software environment. You are encouraged to collaborate with your peers in small groups of 4-5 students in the laboratory. Assessments of projects completed by the students at the end of the corresponding modules will be done on an individual basis (no group assessments). The projects are designed in such a way that the students should be able to complete the projects during the allotted laboratory hours of a given module. Assessments of projects will comprise three components:

1. Assessment of programming ability wherever applicable. This will include both program syntax and program logic development.

2. Assessment of the analysis of the problem and presentation of the solutions. Solutions to questions must be provided in a concise report format using mathematical editing functionalities whenever required.

3. Self and/or peer assessment by students. Rubric for these assessments will be shared with you at the beginning of the respective laboratory class.