Instructions: You must not be in possession of any cheat sheet, notes, or electronic devices like laptops or calculators inside the examination hall. Please answer all **four questions**. Please begin your answer to a given question on a new page. Please show all steps leading to your final answer to receive any credit for your solution. Merely stating the final answer may not fetch you any credit. Maximum point allotted to each question is mentioned in the square bracket on the right margin. Maximum score of this examination is **20**. Total duration of the exam is **2 hours**.

1. Consider the vector space $\mathbb{P}_2$ with bases $B = \{1, x, x^2\}$ and $B' = \{1, 2x, 4x^2 - 2\}$. Answer the following questions.
   (a) Find the change of bases matrices from B to B' and vice-versa. **[3]**
   (b) Consider the linear transformation $T(\phi) = \phi'(x)$ defined for every $\phi \in \mathbb{P}_2$. Find the matrix representation of the transformation $T : \mathbb{P}_2 \to \mathbb{P}_2$ with respect to the basis B. Subsequently, find the matrix representation of T with respect to the basis B'. **[3]**
   (c) Compute the derivative of $\phi(x) = x^2 - \frac{3}{2}x$ using the matrix transformation T. Express ϕ' both as vector coordinates and subsequently, in polynomial form. You must use T in each of the bases B and B' and show that the answer you report is consistent. **[2]**
2. Use the Gauss-Jordan elimination method to invert the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix}$. What is the $null(A)$? Explain your answer. **[5]**
3. Find the basis vectors of the column space of the matrix $B = \begin{pmatrix} 1 & 2 & 2 & -5 & 6 \\ -1 & -2 & -1 & 1 & -1 \\ 4 & 8 & 5 & -8 & 9 \\ 3 & 6 & 1 & 5 & -7 \end{pmatrix}$. What is the dimension of the column space of B? Explain your answer. **[3]**
4. Give an example of a matrix C such that $col(C)$ is the plane with the normal vector $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ in $\mathbb{R}^3$. What is the dimension of the column space of the matrix C? Explain your answer. **[4]**

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