Questions

1. Solve the systems for the variables x_1, x_2, x_3, x_4, and x_5:

   (a)
   $$\begin{aligned} x_2 + 2x_4 + 3x_5 &= 0, \\ 4x_4 + 8x_5 &= 0. \end{aligned}$$

   (b)
   $$\begin{aligned} x_1 + 2x_2 + 2x_4 + 3x_5 &= 0, \\ x_3 + 3x_4 + 2x_5 &= 0, \\ x_3 + 4x_4 - x_5 &= 0, \\ x_5 &= 0. \end{aligned}$$

2. For which values of a, b, c, d, and e is the following matrix in reduced row-echelon form?

   $$A = \begin{bmatrix} 1 & a & b & 3 & 0 & -2 \\ 0 & 0 & c & 1 & d & 3 \\ 0 & e & 0 & 0 & 1 & 1 \end{bmatrix}$$

3. Solve the following system using Gauss Jordan elimination method:

   $$\begin{aligned} 4x_1 + 3x_2 + 2x_3 - x_4 &= 4, \\ 5x_1 + 4x_2 + 3x_3 - x_4 &= 4, \\ -2x_1 - 2x_2 - x_3 + 2x_4 &= -3, \\ 11x_1 + 6x_2 + 4x_3 + x_4 &= 11. \end{aligned}$$

4. Consider the system of equations:

   $$\begin{aligned} x + 2y + 3z &= 4, \\ x + ky + 4z &= 6, \\ x + 2y + (k+2)z &= 6, \end{aligned}$$

   where k is an arbitrary constant.

   (a) For which values of the constant k does this system have a unique solution?

   (b) When is there no solution?

   (c) When are there infinitely many solutions?