

Computational Linear Algebra: FM 126
Phenomenology assessment for section L1

Total time: 1 hour
May 06, 2025

Full Name: _____

UID: _____

Instructions: You must **not** be in possession of any electronic devices like laptops, calculators, phones, and GPT-services while responding to this multiple-choice questionnaire. Answer **all the multiple-choice questions** by circling the correct option. **If you mark more than one option as your answer to any question, your response will be treated as incorrect (even if one of the opted answers is the correct answer).** A maximum of 1 point can be scored in each question. **Maximum score is 10.**

=====START OF QUESTIONS=====

Disclaimer: The plot presented here is a work of fiction. Any violation of physical laws like the conservation of mass and energy should be considered irrelevant during this assessment.

In the hallowed halls of her secluded laboratory, our champion alchemist Esmeira tirelessly pursued the magnum opus: the creation of the Philosopher's Stone (the fabled key from antiquity said to transform lead into gold) and, more profoundly, the formulation of the legendary *Elixir of Life* - granting immortal vitality. Centuries of arcane knowledge, chemical reactions, and celestial alignments had led her to a pivotal discovery, a complex substance, the nascent form of the *Elixir*, which seemed to possess two crucial, intertwined vital essences. She labeled these essences *Vita Corpora* (V) and *Spiritus Fundamentum* (S). These two essences had the distinction of enabling human somata (body cells) to mirror their functional behavior when the Elixir was consumed in appropriate proportion. The Elixir performed better than all other known anti-ageing potions because Esmeira observed that the potency of her formulation grew in a compounded-exponential (super-exponential) fashion when compared with other competing formulations that exhibited a normal exponential growth. A polymath of her age, she believed the universe's secrets were encoded in the language of numbers. She observed how the potential vitality within the Elixir's components changed with each alchemical cycle of refinement, denoted by the stage number k . She meticulously recorded her observations and, to her astonishment, found the changes could be described by a mathematical system.

- Based on the essences *Vita Corpora* (V) and *Spiritus Fundamentum* (S) depicted in the movie in blue and red, respectively, what do you expect the functional behavior of V and S to be with successive refinement stages? Here V_k and S_k are the measured potency of *Vita Corpora* and *Spiritus Fundamentum* after the k^{th} refinement stage.
 - V_k and S_k increase with successive refinement stages $k = 0, 1, 2, \dots$ without oscillations
 - V_k and S_k decrease with successive refinement stages $k = 0, 1, 2, \dots$
 - V_k and S_k remain constant over successive refinement stages $k = 0, 1, 2, \dots$
 - V_k increases and S_k decreases with successive refinement stages $k = 0, 1, 2, \dots$ with small fluctuations
 - V_k and S_k increase over successive refinement stages $k = 0, 1, 2, \dots$ with oscillations
- Given that both the essences V and S are vehicles of somatic growth, Esmeira was able to conjecture the existence of a universal *Rejuvenation Factor* θ based on a new mathematical model. Identify which one of the following mathematical models best describes the successive stages of the aforementioned alchemical refinement necessary for attaining immortality and is best suited to estimate θ appropriately (cf. question 1 to deduce the model from the observation). Note that Esmeria began her alchemical reactions with unit potency of each constituent of vitality.

A.

$$\begin{aligned} V_{k+1} &= S_k \\ S_{k+1} &= V_k \end{aligned}$$

B.

$$\begin{aligned} V_{k+1} &= 0.25V_k \\ S_{k+1} &= -0.25V_k - 0.25S_k \end{aligned}$$

C.

$$\begin{aligned} V_{k+1} &= 1.1V_k + S_k \\ S_{k+1} &= 1.1S_k \end{aligned}$$

D.

$$\begin{aligned} V_{k+1} &= 0.9V_k \\ S_{k+1} &= 1.1V_k + 0.9S_k \end{aligned}$$

E.

$$\begin{aligned} V_{k+1} &= 1.1S_k \\ S_{k+1} &= -0.75V_k \end{aligned}$$

3. Based on your response to question 1, identify the type of the mathematical model.

- A. *Linear model*
- B. Non-linear model
- C. Continuum mechanics model
- D. Random matrix model
- E. stochastic model with unpredictable and random outcomes

4. When the correct mathematical system, identified in question 1, is written as a matrix-vector equation of the form $\begin{pmatrix} V_{k+1} \\ S_{k+1} \end{pmatrix} = M \begin{pmatrix} V_k \\ S_k \end{pmatrix}$, the *matrix of vitality transformation*, M is a 2×2 matrix. Identify the correct M .

- A. $M = \begin{pmatrix} 1.1 & 1 \\ 0 & 1.1 \end{pmatrix}$
- B. $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- C. $M = \begin{pmatrix} 0.25 & 0.25 \\ -0.25 & -0.25 \end{pmatrix}$
- D. $M = \begin{pmatrix} 0 & 1.1 \\ -1.1 & 0 \end{pmatrix}$
- E. $M = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

5. The algebraic multiplicity (AM) and the geometric multiplicity (GM) of the appropriate entity related to the correct transformation matrix are the following.

- A. $AM = 2, GM = 1$
- B. $AM = GM = 2$
- C. $AM = GM = 1$
- D. $AM = 1, GM = 2$
- E. $AM = 2, GM$ is indeterminate

6. Is the matrix M diagonalizable? Why?

- A. *No. There is only one linearly independent eigenvector. M cannot be diagonalized using a similarity transformation.*
- B. Yes. M is invertible, therefore it can be diagonalized using a similarity transformation of the form MDM^{-1} .
- C. Yes. M has a full range of linearly independent eigenvectors.

- D. No. $GM = 1$ because M has only one eigenvalue.
- E. No. Determinant of M is not equal to zero.
7. What is the most appropriate *Rejuvenation Factor* θ for the alchemical system? The θ must be a marker for the efficacy of the Elixir.
- A. The maximum eigenvalue of M
- B. The determinant of M
- C. The trace of M
- D. The maximum eigenvalue of $M^T M$
- E. The dominant eigenvalue of M
8. Based on the correct choice of the model in question 2, deduce the analytic form of the solution for the potency variables V_k and S_k .
- A. $V_k = V_0(1.1)^k + S_0 k(1.1)^{k-1}$ and $S_k = S_0(1.1)^k$
- B. $V_k = V_0(0.9)^k$ and $S_k = V_0(0.9)^k + S_0 k(1.1)^{-k}$
- C. $V_k = V_0(0.25)^k + S_0(0.25)^k$ and $S_k = V_0(0.25)^{-k} - S_0(0.25)^{-k}$
- D. $V_k = V_0(1.1)^k + S_0 k(1.1)^{k-1}$ and $S_k = S_0 k(1.1)^k$
- E. $V_k = 1.1V_0 k$ and $S_k = -0.75S_0 k$
- F. None of the options
9. Which one of the following linear algebra concepts may be used to correctly diagnose the observed functional behavior of the potency of the Elixir?
- A. $GM < AM$ of the matrix model. This means the matrix does not have a full set of eigenvectors and is therefore defective (non-diagonalizable). This is responsible for the super-exponential growth of the respective potency metrics V_k and S_k over successive refinement stages k .
- B. $GM = AM = 2$ of the matrix model. This means the matrix has a full set of eigenvectors and the solution for V_k and S_k exhibit an exponential growth with refinement cycle k .
- C. $GM = 1$, $AM = 2$ of the matrix model. The matrix is invertible and the inverse is equal to its transpose. This means the matrix is orthogonal and its eigenvectors are orthogonal and independent of each other. Therefore, the potency metrics V_k and S_k change independent of each other over successive refinement stages k .
- D. $AM = GM = 1 < \text{rank}(M)$. This means the potency metrics V_k and S_k exhibit a damped evolution over k and the Elixir was a fatally dangerous poison. Police raided the castle. Esmeira's whereabouts are unknown since the raid.
- E. The eigenvalues of M are zeros because the diagonal entries of M are zeros. The Elixir stunts life and individuals die instantly after consumption. Esmeira was arrested and imprisoned.
10. If a customer purchased the Elixir from Esmeira in large amounts, should you expect the person to have actually become immortal by prolonged consumption of the potion?
- A. Yes. The Elixir was effective, the model bears linear-exponential growth modes that are mimicked by human somatic cells. Therefore, immortality was granted by sustained use of the potion that the customer had purchased from Esmeira.
- B. No. The compounded-growth response of the Elixir is a linear-exponential function of the refinement stage k . After a while, the efficacy of the Elixir diminished due to adulteration. The potion was never refined again by Esmeira.
- C. Yes. The Rejuvenation Factor $\theta = 1$ guaranteed unity with divinity. Therefore, the customer became immortal.
- D. No. The Rejuvenation Factor is $\theta < 1$. Esmeira was a con artist and her potion was useless.
- E. Not knowable. The Rejuvenation factor θ cannot be determined from the model equations.

=====END OF QUESTIONS=====