Mathematics of Uncertainty: FM 122 18th April, 2022

Mid-semester examination Total time: 1 hour 30 min

Instructions: You must **not** be in possession of any cheat sheet, notes, or electronic devices like laptops or calculators inside the examination hall. **Please answer all three questions.** Please begin your answer to a given question on a new page. Please **show all steps** leading to your final answer to receive any credit for your solution. Merely stating the final answer may not fetch you any credit. Maximum point allotted to each question is mentioned in the square bracket on the right margin. **Maximum score is 30.**

========== **START OF QUESTIONS** ==========

1. The $\mathfrak{M}$-09 is a twin engine jet. Let X be the random time to failure of engine-1 and Y be the random time to failure of engine-2. X and $Y$are independent random variables with distribution $exp(\mu_1)$ and $exp(\mu_2)$ with mean times to failure $\frac{1}{\mu_1} = \frac{1}{\mu_2} = 100$ flying hours. What is the probability that there is a dual engine flame out in more than 75 flying hours? **[10]**

2. Consider a population of tumour cells where each cell has a random number of progeny. Consider that the number of progeny of the proliferating cells is i.i.d. with mean μ. Suppose the process starts with one cell in generation zero. For simplicity let us assume there are no deaths. Calculate the expected total number of tumour cells in n generations? For $n \to \infty$, find a condition for arresting the rate of growth of tumour cells. **[10]**

   Hint: In the formulation of the question above, consider $T_k \equiv S$ = number of cells in generation k. $T_k = X_1 + X_2 + \ldots + X_{T_{k-1}}$. Here $X_1, X_2, etc \ldots$. are the numbers of progeny of the first, second, etc. ... cells in generation $k-1$.

3. Consider m balls are placed one after the other into one of b bins. The bin for each ball is selected at random. Answer the following two questions.
   - What is the expected value of the number of empty bins?
   - Give an estimate for your answer above when b is very large (i.e. $b \to \infty$). **[10]**

Hint: In order to solve this problem, it may be more convenient to use an indicator random variable.

========== **END OF QUESTIONS** ==========