Worksheet (WL5.1)

Orthogonal basis, properties of orthonormal vectors, orthogonal projection and orthogonal complement, properties of orthogonal complement, advantage of orthogonal transformations, Gram-Schmidt process

Name and section: ____________________

Instructor's name: ____________________

1. Find the orthonormal basis $\vec{u_1}, \vec{u_2}$ of the subspace $V = span\left(\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\9\\9\\1\end{bmatrix}\right)$ of $\mathbb{R}^4$, with basis $\vec{v_1} = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$, $\vec{v_2} = \begin{bmatrix}1\\9\\9\\1\end{bmatrix}$

2. Using Gram-Schmidt process, find the orthonormal basis corresponding the basis
$$\mathbb{B} = \{(1,7,1,7),\ (0,7,2,7),\ (1,8,1,6)\}$$

3. Find an orthonormal basis of the plane
$$x_1 + x_2 + x_3 = 0$$
.

4. Find an orthonormal basis of the kernel of the matrix
$$A = \begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 2 & 3 & 4\end{bmatrix}$$

5. Consider the vector
$$\vec{v} = \begin{bmatrix}1\\2\\3\\4\end{bmatrix} \in \mathbb{R}^4$$
Find the basis of the subspace of $\mathbb{R}^4$ consisting of all vectors perpendicular to $\vec{v}$.

6. Find the orthogonal projection of $\begin{bmatrix}9\\0\\0\\0\end{bmatrix}$ onto the subspace of $\mathbb{R}^4$ spanned by
$$\begin{bmatrix}2\\2\\1\\0\end{bmatrix} \text{ and } \begin{bmatrix}-2\\2\\0\\1\end{bmatrix}$$

7. Find the QR factorization of the matrix
$$A = \begin{bmatrix}0 & -3 & 0\\ 0 & 0 & 0\\ 2 & 0 & 0\\ 0 & 0 & 4\end{bmatrix}.$$