Tutorial Worksheet-2 (WL2.2, WL3.1)
Define vector spaces of m×n matrices and its practical applications, introduction to system of linear equations, Row-Reduced Echelon form, rank of a matrix, linear transformation

Name and section: _____

Instructor's name: _____

1. Check the consistency of the following system of equations graphically:

$$\begin{aligned} x - 3y &= 4 \\ -2x + 6y &= 5 \end{aligned}$$

2. Prove that the set of Matrices of order 2×3 denoted as $\mathbb{M}_{2\times 3}(\mathbb{R})$ forms a vector space over $\mathbb{R}$ under usual addition and scalar multiplication of matrices.

3. For which values of the constant c is $\begin{bmatrix} 1 \\ c \\ c^2 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$.

4. Convert the following matrices into the rref

$$\begin{bmatrix} 2 & 4 & 10 & -18 \\ -1 & -2 & -1 & 3 \\ -2 & -3 & 0 & 3 \\ 1 & 1 & -1 & -5 \end{bmatrix}, \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

5. Evaluate the rank of matrices which gives in the problem (5).

6. Reduce the following matrix into rref (Row-Reduced Echelon form) and find its rank

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 7 & 14 \end{bmatrix}$$

   Also list the pivotal elements of the matrix.

7. Consider the transformation T from $\mathbb{R}^2$ to $\mathbb{R}^3$ given by

$$T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

   Is this transformation linear. If so, find its matrix representation.