Computational Linear Algebra: FM 126 — Total time: 1 hour

Quiz-2 for section: L2 — May 01, 2025

Full Name: ______________________ **UID:** ______________________

Instructions: You must **not** be in possession of any cheat sheet, notes, or electronic devices like laptops or calculators inside the examination hall. Answer **all ten multiple-choice questions (MCQs)**. The score allotted to each question is **one**. There will be a penalty of **0.25 marks** for each wrong answer. ***If you mark more than one option as your answer to any question, your response will be treated as incorrect and the penalty will apply (even if one of the opted answers is the correct answer).*** Darken the circle against the correct option. **Maximum score is 10**.

========================START OF QUESTIONS========================

1. What are the eigenvalues of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$?
   - ○ 6,1
   - √ *5,2*
   - ○ 3,4
   - ○ 2,2

2. Which of the following is an eigenvector of $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ with eigenvalue $\lambda = 3$?

   ○ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ○ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ √ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ○ $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

3. The matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is diagonalizable. Why or why not?
   - ○ Yes, because the determinant is non zero.
   - ○ Yes, because it has two distinct eigenvalues.
   - ○ No, because it has only one eigenvalue and is not diagonalizable.
   - √ *No, because it has only one linearly independent eigenvector.*

4. Which matrix is *not* diagonalizable?

   ○ $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ ○ $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ ○ $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ √ $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

5. The power method will converge to:
   - √ *The eigenvector associated with the largest (in magnitude) eigenvalue*
   - ○ A random eigenvector
   - ○ The eigenvector associated with the smallest (in magnitude) eigenvalue
   - ○ The inverse of the matrix

6. Start with $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and apply the power method once to $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. What is $x_1 = Ax_0$?
   - ○ $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
   - ○ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
   - √ $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$
   - ○ $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

7. What is the projection of the vector $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ onto the vector $\vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$?

   ○ $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ √ $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ ○ $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ ○ $\begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$

8. Let $\vec{u} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.Then

   - √ *$\vec{u}$ and $\vec{v}$ are orthogonal*
   - ◯ $\vec{u}$ and $\vec{v}$ are orthonormal
   - ◯ $\vec{u}$ and $\vec{v}$ does not span $\mathbb{R}^2$
   - ◯ $\vec{u}$ and $\vec{v}$ are linearly dependent

9. Which condition ensures a matrix is diagonalizable?

   - √ *The matrix has distinct eigenvalues*
   - ◯ Matrix is an invertible matrix
   - ◯ A matrix having determinant zero
   - ◯ A matrix having trace zero

10. What is the characteristic polynomial of $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$?

   - ◯ $\lambda^2 - 2\lambda + 3$
   - ◯ $\lambda^2 - 2\lambda - 1$
   - ◯ $\lambda^2 - 2\lambda - 4$
   - √ $\lambda^2 - 2\lambda - 3$