Computational Linear Algebra: FM 126
Quiz-2 for section: L1

Total time: 1 hour
May 01, 2025

Full Name: ______________________ **UID:** ______________________

Instructions: You must **not** be in possession of any cheat sheet, notes, or electronic devices like laptops or calculators inside the examination hall. Answer **all ten multiple-choice questions (MCQs)**. The score allotted to each question is **one**. There will be a penalty of **0.25 marks** for each wrong answer. ***If you mark more than one option as your answer to any question, your response will be treated as incorrect and the penalty will apply (even if one of the opted answers is the correct answer).*** Darken the circle against the correct option. **Maximum score is 10**.

========================START OF QUESTIONS========================

1. What are the eigenvalues of the matrix $A = \begin{bmatrix} -2 & -4 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$?
   - ○ 0,1,2
   - ○ 0,-2,0
   - ○ 1,3,-1
   - √ *1,2,1*

2. The eigenvectors of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$ corresponding to different eigenvalues are:
   - ○ linearly dependent
   - ○ parallel
   - √ *linearly independent*
   - ○ not orthogonal

3. For the matrix $A = \begin{bmatrix} 4 & -3 \\ 5 & -4 \end{bmatrix}$, suppose $A = PDP^{-1}$. Which of the following pairs (P, D) is correct?
   - ○ $P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$
   - ○ $P = \begin{bmatrix} 1 & 1 \\ -5 & -3 \end{bmatrix}, D = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
   - ○ $P = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
   - √ $P = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

4. For the matrix $A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$, 2 is an eigenvalue of A with
   - √ *algebraic multiplicity 2 and geometric multiplicity 1.*
   - ○ algebraic multiplicity 1 and geometric multiplicity 2.
   - ○ algebraic multiplicity 1 and geometric multiplicity 1.
   - ○ algebraic multiplicity 2 and geometric multiplicity 2.

5. What is the first step in the power method?
   - √ *Start with an appropriate initial guess vector x_0 then compute $x_1 = Ax_0$*
   - ○ Normalize the matrix
   - ○ Solve $A\vec{x} = \lambda\vec{x}$
   - ○ Take the inverse of the matrix

6. Start with $x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and apply the power method twice to $A = \begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix}$. What is $x_2 = A^2 x_0$?

   ○ $\begin{bmatrix} 14 \\ 16 \end{bmatrix}$

   ○ $\begin{bmatrix} 12 \\ 14 \end{bmatrix}$

   √ $\begin{bmatrix} 16 \\ 14 \end{bmatrix}$

   ○ $\begin{bmatrix} -14 \\ 15 \end{bmatrix}$

7. Let $A_{2\times 2}$ have eigenvalues 3 and 5. Which of the following is a diagonal matrix D such that $A = PDP^{-1}$?

   ○ $D = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

   ○ $D = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$

   √ $D = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$

   ○ $D = \begin{bmatrix} 0 & 3 \\ 0 & 5 \end{bmatrix}$

8. Which of the following formulas correctly gives the projection of $\vec{v}$ onto $\vec{u}$?

   ○ $\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{u}$

   ○ $\frac{\vec{v} \cdot \vec{v}}{\vec{u} \cdot \vec{u}} \vec{u}$

   ○ $\frac{\vec{v} \cdot \vec{v}}{\vec{u} \cdot \vec{u}} \vec{v}$

   ○ $\vec{v} - \vec{u}$

9. In the QR decomposition, R is always

   √ *an upper triangular matrix*

   ○ a lower triangular matrix

   ○ a zero matrix

   ○ a diagonal matrix

10. For the matrix $A = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}$, the Q matrix in its QR decomposition is $Q = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$. What is the corresponding R matrix?

   ○ $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{6} \end{bmatrix}$

   √ $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & 2\sqrt{2} \end{bmatrix}$

   ○ $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & 2 \end{bmatrix}$

   ○ $\begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{2} \end{bmatrix}$